Quantum: The Secret of Cohesion: How Waves Hold Matter Together

←Quantum

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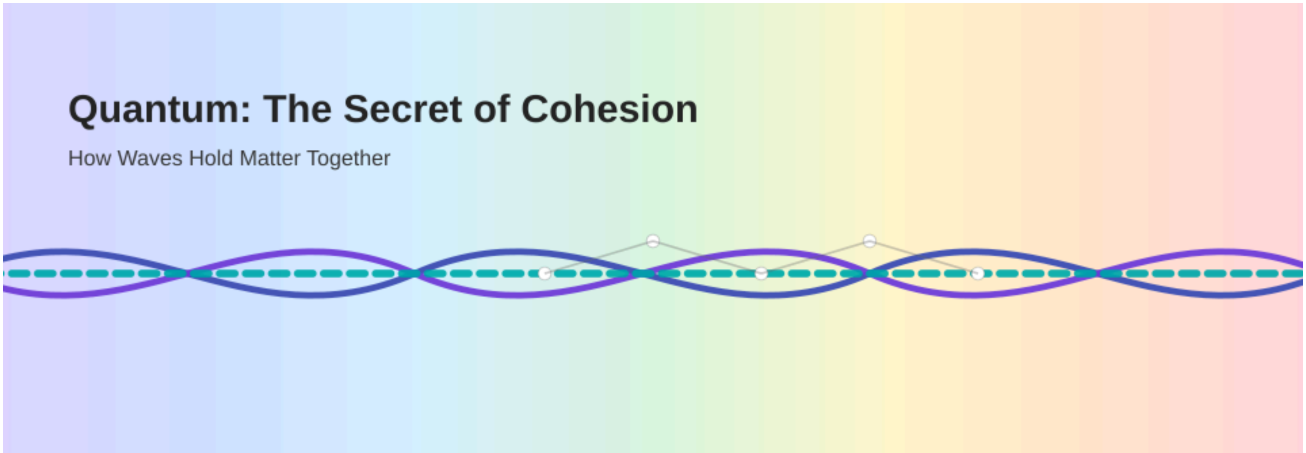
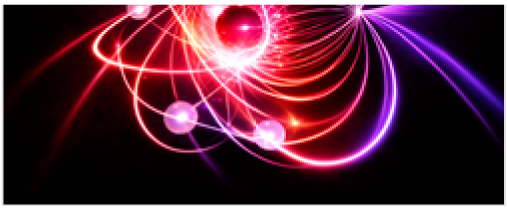


Overview: Quantum Mechanisms of Cohesion

Classical vs Quantum Explanation for Matter Stability

Opposite charges attract, but that explanation cannot account for the stability of atoms.
If electrons were point particles, they would collapse into the nucleus.

The stability of matter is quantum mechanical.
Electrons are described by wavefunctions.
These waves:



Artistic impression of an atom 9

Waves do not hold matter together by acting like glue. Instead, cohesion arises from the overlap, interference, and symmetry constraints of quantum wavefunctions, together with the balance between kinetic energy and Coulomb forces.

In this course we examine how:

- The Schrödinger equation already encodes stability
- Wavefunction overlap produces chemical bonds
- Fermi statistics prevents collapse
- Collective coherence gives rise to metals and superconductors
- Decoherence in open systems can modify binding

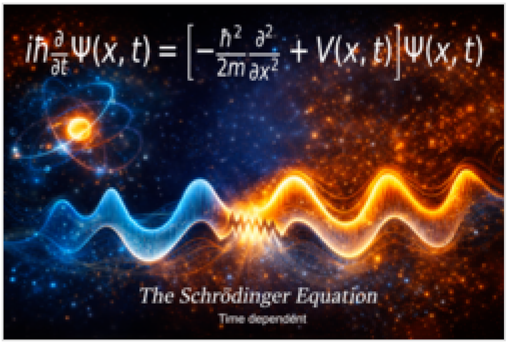
Matter holds together because particles behave as quantum waves.

Classical vs Quantum Picture.

- Cohesion in matter exists because particles have wave character, and **wavefunctions overlap, interfere, and obey symmetry constraints.**

Let's build this step by step.

1️⃣ Starting Point: The Schrödinger Equation



Schrodinger equation time dependant

The **Schrödinger equation** a partial differential equation that overlooks the wave function of non-relativistic quantum-mechanical systems.[1]:1–2 Its discovery was significant in the development of quantum mechanics. It is named after Erwin Schrödinger, who postulated the equation in 1925 and published it in 1926. It forms the basis for the work that leaded to his Nobel Prize in Physics in 1933.[2][3][4]

For an electron in a potential $V(\mathbf{r})$:

$$i\hbar\frac{\partial\psi(\mathbf{r},t)}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r})\right)\psi(\mathbf{r},t)$$

The first term,

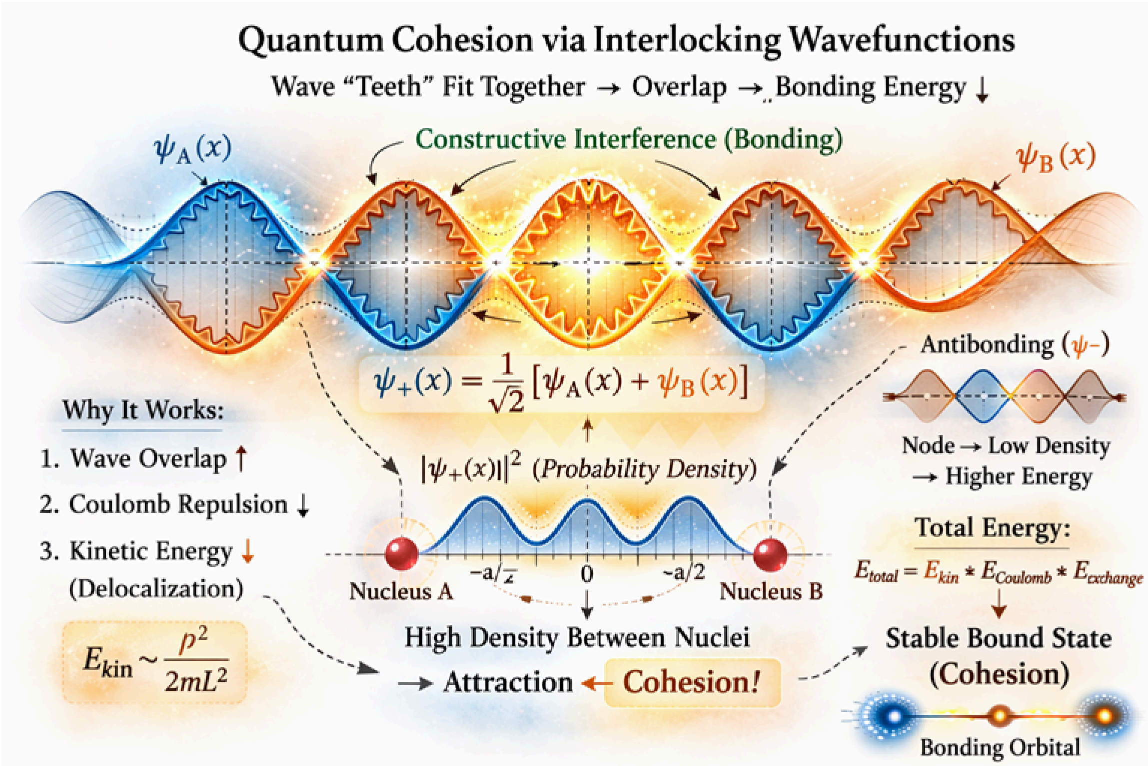
$$-\frac{\hbar^2}{2m}\nabla^2\psi$$

$$V(\mathbf{r})\psi$$

represents the interaction energy (e.g., the Coulomb potential).

Cohesion emerges from the interplay between:

- Wave-like delocalization (kinetic term $-\frac{\hbar^2}{2m}\nabla^2$)
- Coulomb attraction between charges encoded in $V(\mathbf{r})$



2️⃣ Wave Nature of Particles and Quantum Stability

Matter waves, a central part of the theory of quantum mechanics, being half of wave–particle duality. All scales where measurements have been practical, matter exhibits wave-like behavior. A beam of electrons can be diffracted like a beam of light or a water wave.

Matter behaves like a wave was proposed by French physicist Louis de Broglie in 1924, and matter waves are also known as **de Broglie waves**.

If electrons were classical point particles, they would collapse into the nucleus and atoms would not be stable.

To find the wavelength equivalent to a moving body, de Broglie set the total energy from special relativity for that body equal to *hv*:

(Modern physics no longer uses this form of the total energy; the W:energy–momentum relation:energy–momentum relation has proven more useful.) De Broglie identified the velocity of the particle, v, with the wave group velocity in free space:

$$v_{\rm g} \equiv \frac{\partial \omega}{\partial k} = \frac{d\nu}{d(1/\lambda)}$$

(The modern definition of group velocity uses angular frequency ω and wave number k). By applying the differentials to the energy equation and identifying the relativistic momentum:

$$p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}}$$

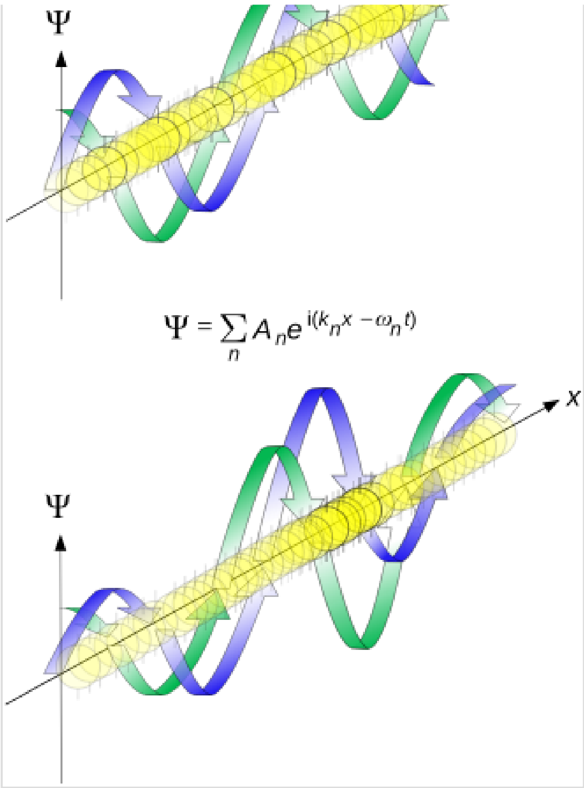
then integrating, de Broglie arrived at his formula for the relationship between the wavelength, λ, associated with an electron and the modulus of its momentum, p, through the Planck constant, h:[5]

$$\lambda = \frac{h}{p}.$$

The quantum kinetic energy associated with spatial confinement scales as

$$E_{\rm kin} \sim \frac{\hbar^2}{2mL^2}$$

where L is the characteristic localization length.



Propagation of **de Broglie waves** in one dimension – real part of the complex amplitude is blue, imaginary part is green. The probability (shown as the color opacity) of finding the particle at a given point x is spread out like a waveform; there is no definite position of the particle. As the amplitude increases above zero the slope decreases, so the amplitude diminishes again, and vice versa. The result is an alternating amplitude: a wave. Top: plane wave. Bottom: wave packet.

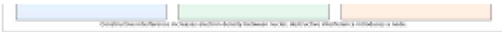
As L becomes smaller (stronger localization), the kinetic energy increases rapidly. This follows from the Laplacian term in the Schrödinger equation and is closely related to the uncertainty principle,

$$\Delta x \, \Delta p \geq \frac{\hbar}{2}$$

Thus, wave behavior prevents atomic collapse.

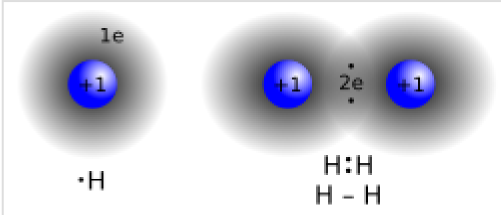
3️⃣ Covalent Bonding and Wavefunction Overlap

Covalent bonding a chemical bond that involves electrons to form electron pairs between atoms. These electron pairs are known as **shared pairs** or **bonding pairs**. Stable balance of attracting and repulsive forces between atoms, when they share electrons, is known as covalent bonding.[6] For molecules, to



H2 molecular orbitals

Consider two hydrogen atoms. Their atomic orbitals can combine into symmetric and antisymmetric superpositions:



A covalent bond forming H_2 (right) where two hydrogen atoms share the two electrons

Bonding combination:

$$\psi_+ = \frac{1}{\sqrt{2}}(\psi_A + \psi_B)$$

Antibonding combination:

$$\psi_- = \frac{1}{\sqrt{2}}(\psi_A - \psi_B)$$

The bonding state ψ_+ has increased probability density between the nuclei:

$$|\psi_+|^2 > |\psi_A|^2 + |\psi_B|^2$$

in the internuclear region.

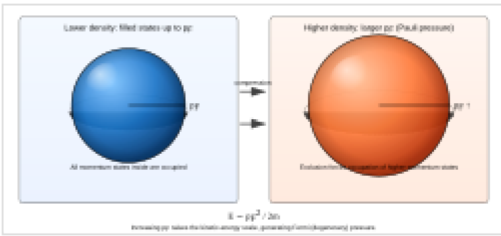
This lowers the total energy through:

- Electron delocalization
- Reduced Coulomb repulsion
- Lower effective kinetic energy due to spreading

Bonding is therefore an energetic consequence of constructive wave interference.

4️⃣ Fermi Statistics and Pauli Effects

A degenerate Fermi gas represents a state of matter where particles (fermions) are packed at high density and low temperatures, causing quantum effects. The pressure of a degenerate Fermi gas, known as degeneracy pressure, differs between high-density (high-pressure) and low-density (low-pressure) regimes, because the pressure depends on density (n) rather than temperature (T)
Electrons are fermions, so the total wavefunction must be antisymmetric under exchange:



Fermi sphere in momentum space illustrating Pauli exclusion. States are filled up to the Fermi momentum p_F; increasing density enlarges p_F, raising kinetic energy and generating Fermi pressure.

$$\Psi(\mathbf{r}_1, \mathbf{r}_2) = -\Psi(\mathbf{r}_2, \mathbf{r}_1)$$

This implies the Pauli exclusion principle:

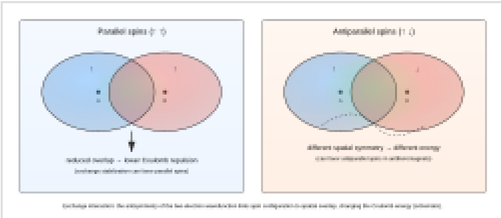
$$n_i \leq 1$$

$$E \sim \frac{p_F^2}{2m}$$

This quantum pressure stabilizes matter and prevents collapse.

5️⃣ Exchange Interaction and Collective Order

Antisymmetry, the total energy includes an exchange term. In simplified form, the exchange contribution between two states can be written as



Exchange interaction: antisymmetric $\boldsymbol{\Psi}(\mathbf{r}_1, \mathbf{r}_2)$ couples spin configuration to spatial overlap, changing Coulomb energy and allowing ferro- or antiferromagnetic ordering.

$$E_{\text{ex}} \sim \int\int \psi_i^*(\mathbf{r}_1)\psi_j^*(\mathbf{r}_2)\frac{e^2}{|\mathbf{r}_1 - \mathbf{r}_2|}\psi_i(\mathbf{r}_2)\psi_j(\mathbf{r}_1)\, d\mathbf{r}_1\, d\mathbf{r}_2$$

This term has no classical analogue and arises purely from wavefunction symmetry.

It explains phenomena such as:

- Ferromagnetism
- Antiferromagnetism
- Additional molecular stabilization

6️⃣ Metallic Bonding: Bloch Waves

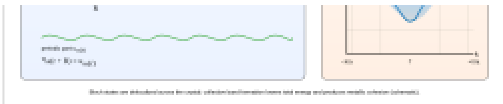
Metallic bonding is a chemical bonding that arises from the electrostatic attractive force between conduction electrons (in the form of an electron cloud of delocalized electrons) and positively charged metal ions. It is the sharing of *free* electrons among a crystal structure of positively charged ions (cations). Metallic bonding accounts for physical properties of metals, such as strength, ductility, thermal and W:electrical resistivity and conductivity, opacity, and lustre.[7][8][9][10]

In a periodic lattice potential $V(\mathbf{r})$, electron eigenstates take the Bloch form:

$$\psi_{n\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}}u_{n\mathbf{k}}(\mathbf{r})$$

where

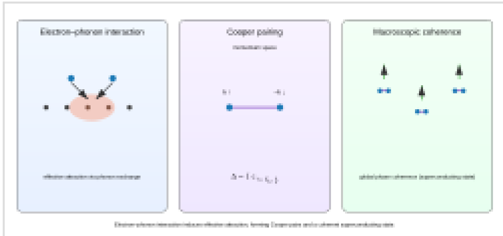
These delocalized wave functions extend across the crystal and lower total energy collectively, producing metallic cohesion.



Bloch wave and band formation in a periodic lattice. States of the form $\psi_{nk}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} u_{nk}(\mathbf{r})$ are delocalized across the crystal and give rise to metallic energy bands.

7️⃣ Superconductivity as a Cohesive Quantum Phenomenon

Superconductivity, physical properties observed in **superconductors**: materials where electrical resistance vanishes and magnetic fields are expelled from the material. Unlike a standard metallic conductor, whose resistance decreases gradually as its temperature is lowered, even down to near absolute zero, a superconductor characteristics are critical temperature below which the resistance drops abruptly to zero.[11][12] An electric current through a loop of superconducting wire can persist indefinitely with no power source.[13][14][15][16]



Electron–phonon interaction induces an effective attraction, forming Cooper pairs. A macroscopic coherent state emerges with the order parameter $\Delta \sim \langle c_{-k\downarrow} c_{k\uparrow} \rangle$.

Through electron-phonon interaction, an effective attractive interaction can arise:

$$H_{\text{int}} = \sum_{k,k'} V_{kk'} c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} c_{-k'\downarrow} c_{k'\uparrow}$$

This leads to Cooper pairing and a macroscopic coherent state with order parameter

$$\Delta \sim \langle c_{-k\downarrow} c_{k\uparrow} \rangle$$

Cohesion here emerges from collective quantum phase coherence.

8️⃣ Stability of Matter: Mathematical Perspective

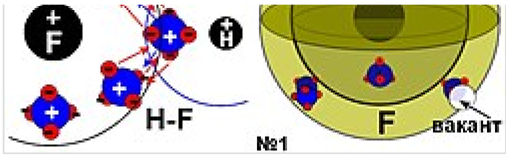
Stability of matter refers to the ability of a large number of charged particles, such as electrons and protons, to create macroscopic objects without collapsing or blowing apart due to electromagnetic interactions. Classical physics predicts that such systems should be inherently unstable due to attractive and repulsive electrostatic forces between charges, and thus the stability of matter was a theoretical problem that required a quantum mechanical explanation.

The first solution to this problem was provided by Freeman Dyson and Andrew Lenard in Freeman Dyson 1967–1968,[17][18] but a shorter and more conceptual proof was found later by Elliott Lieb in 1975 using the Lieb–Thirring inequality.[19] The stability of matter is partly due to the uncertainty principle

$$E_{\text{total}} = T_{\text{quantum}} + V_{\text{Coulomb}}$$

The kinetic term originates from wave curvature and the Coulomb term from electromagnetic interaction.

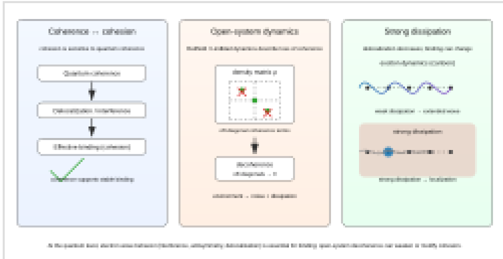
Without the quantum kinetic term, the energy would decrease without bound and matter would collapse.



Atom model. The structure of the atom. Static theory. №1 Series 12 photos.

9️⃣ Decoherence Effects on Binding in Open Quantum Systems.

In physics, an **open quantum system** is a quantum mechanical system that interacts with an external quantum system, "the *environment*" or a *"bath"*. In general, interactions significantly change the dynamics of the system, the information contained in the system is then lost to its environment. No quantum system is completely isolated from its surroundings,[21] it is important to develop a theoretical framework for treating these interactions to obtain an accurate understanding of quantum systems.



Open quantum systems: Redfield/Lindblad decoherence suppresses off-diagonal terms of ρ, reducing delocalization and potentially changing effective binding; cohesion is sensitive to coherence.

In open systems:

- Cohesion is related to coherence.
- Redfield/Lindblad dynamics describe loss of coherence.
- Decoherence can influence binding (e.g., exciton dynamics).

Under strong dissipation:

- Delocalization decreases
- Effective binding can change

So:

- Cohesion is sensitive to quantum coherence.

🔬 Summary: What Holds Matter Together?

Do waves in matter create cohesion?

Not as classical vibrations.

But at the quantum level:

- Yes — the wave character of electrons is essential for binding.

See also

- Quantum
- Quantum A Matter Of Size
- Quantum A Spooky Action at a Distance
- Quantum: A Walk Through the Universe
- Number of independent spatial modes in a spherical volume
- Quantum Computing Algorithms in the NISQ Era
- Quantum Formulas Collection
- Quantum Matter Elements and Particles
- Quantum mechanics
- Quantum mechanics/Timeline
- Quantum mechanics measurements
- Quantum Noisy Qubits
- Quantum optics beam splitter experiments
- Quantum: The Secret of Cohesion: How Waves Hold Matter Together
- Quantum Ultra fast lasers
- Template:Quantum optics operators
- Physical Sciences

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